Topology and its Applications 38 (1991) 147-154
North-Holland

Approximate resolutions and covering dimension*

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Rececived 1 May 1989
Revised 6 November 1989

Abstract

Watanabe, T., Approximate resolutions and covering dimension, Topology and its Applications 38 (1991) 147-154.

It is proven that a topological space X has covering dimension $\dim X \leq n$ if and only if X admits an approximate resolution of polyhedra X_a with $\dim X_a \leq n$.

Keywords: Approximate inverse system, approximate resolution, covering dimension.

AMS (MOS) Subj. Class.: Primary 54B25, 54F45, 54C56.

Introduction

Compact metric spaces X are often defined and studied by expressing X as the inverse limit of an inverse sequence of compact polyhedra or compact ANR's. This idea is fruitful to investigate compact metric spaces. For example, in 1937, Freudenthal [2] proved that

(i) Every compact metrizable space X is an inverse limit of an inverse sequence of finite polyhedra P_i, whose covering dimension $\dim P_i \leq \dim X$.

Although, inverse systems and their limits are also defined for arbitrary spaces, many theorems, which hold for compact metric spaces, fail for more general spaces. Therefore, in 1981 Mardešić [5] introduced the notion of a resolution of a space and proved that

(ii) Every space X admits a polyhedral resolution and also an ANR-resolution (also see [1]).

* Dedicated to Professor Ryosuke Nakagawa on his sixtieth birthday.

If X is topologically complete (i.e., admits a complete uniform structure), resolutions of X are at the same time inverse limits, but the converse does not hold. Therefore, one can view the notion of resolution as a suitable restriction of the notion of inverse limit. In the compact case the two notions coincide and the theory of compact inverse limits extends to resolutions.

It was noticed long ago that the theory of inverse systems of polyhedra in the compact non-metric case also has some defects. For instance, Mardešić [3] and Pasynkov [13] showed that (i) does not hold for compact Hausdorff spaces. That is,

(iii) There is a 1-dimensional compact space M which is not the inverse limit of any inverse systems of finite polyhedra P_a with $\dim P_a \leq 1$.

For this and other reasons Mardešić and Rubin [6] have recently introduced the notion of approximate inverse system of metric compacta and they showed that

(iv) A compact Hausdorff space X has dimension $\dim X \leq n$ if and only if X admits an approximate inverse system of compact polyhedra X_a with $\dim X_a \leq n$.

Recently Mardešić and Watanabe [11] introduced the notions of an approximate inverse system and an approximate resolution for arbitrary spaces. Note that the notion of an approximate resolution in the sense of Watanabe [15] coincides with the notion of a commutative approximate resolution in the sense of Mardešić and Watanabe [11].

The purpose of the present paper is to show the following theorem, which generalizes (iv):

Theorem 1. *A topological space X has* $\dim X \leq n$ *if and only if X admits an approximate reslution of polyhedra X_a with* $\dim X_a \leq n$.

Note that the space M in (iii) does not admit any resolution of polyhedra/ANRs K_a with $\dim K_a \leq 1$ (see (iii) and [10]). However, by (ii), M admits a resolution of polyhedra/ANRs P_a. In this case, $\dim P_a \geq 2$ for almost all indexes. This means that the notion of approximate resolution is more natural than the notions of an inverse limit and a resolution.

Proof of Theorem 1

Without any specification we shall use the same terminology and notions as in [11].

Let X be a topological space. Let $\mathrm{Cov}(X)$ be the set of all normal open coverings of X. Let $\mathcal{U}, \mathcal{V} \in \mathrm{Cov}(X)$. We say that $\mathcal{V}$ is a refinement of $\mathcal{U}$, in notation $\mathcal{V} < \mathcal{U}$, provided for any $V \in \mathcal{V}$ there is a $U \in \mathcal{U}$ with $V \subset U$. Let $\mathrm{st}(T, \mathcal{U}) = \bigcup\{U \in \mathcal{U}: T \cap U \neq \emptyset\}$ for a subset $T \subset X$ and $\mathrm{st}\,\mathcal{U} = \{\mathrm{st}(U, \mathcal{U}): U \in \mathcal{U}\}$. Inductively, $\mathrm{st}^n\mathcal{U} = \mathrm{st}(\mathrm{st}^{n-1}\mathcal{U})$ for any integer $n \geq 2$. The order of $\mathcal{U}$, in notation $\mathrm{ord}(\mathcal{U})$, is the largest integer n such that $\mathcal{U}$ contains n elements with non-empty intersection, or ∞ if no such integer exists. We say that $\dim X \leq n$ provided for any $\mathcal{U} \in \mathrm{Cov}(X)$ there is a $\mathcal{V} \in \mathrm{Cov}(X)$ such that $\mathcal{V} < \mathcal{U}$ and $\mathrm{ord}(V) \leq n+1$ (see [12]).

Let X be a space and let $\Lambda = \{f_a: a \in A\}$ be a set of maps $f_a: X \to K_a$. We put $D(\Lambda) = \{K_a: a \in A\}$. Let $\mathcal{P}$ be a set of spaces. We say that Λ is an *approximate semi-projection* of X with respect to $\mathcal{P}$ provided it satisfies the following condition:

(ASP) For any $P \in \mathcal{P}$, any $\mathcal{U} \in \mathrm{Cov}(P)$ and any map $f: X \to P$, there exist an $a \in A$ and a map $h: K_a \to P$ such that $(hf_a, f) < \mathcal{U}$.

The following is easy to show:

Lemma 2. *Let $\mathcal{P}$ and $\mathcal{L}$ be sets of spaces. If $\mathcal{L} <_a \mathcal{P}$ and Λ is an approximate semi-projection of X with respect to $\mathcal{P}$, then so is Λ with respect to $\mathcal{L}$.*

Corollary 3. *Let $\mathcal{P} =_a \mathcal{L}$. Λ is an approximate semi-projection of X with respect to $\mathcal{P}$ if and only if so is Λ with respect to $\mathcal{L}$.*

We say that Λ is an *approximate semi-projection* of X provided $D(\Lambda) \subset$ **APOL** and Λ is an approximate semi-projection with respect to **POL**. Here **APOL**, **POL** and **ANR** are the collections of all approximate polyhedra, all polyhedra, and all ANRs, respectively. Since **POL** $=_a$ **ANR** by (2.4) of [11], we have that

Corollary 4. *Λ is an approximate semi-projection of X if and only if so is Λ with respect to* **ANR**.

Theorem 5. *Let X be a topological space and Λ be a collection of maps $f_a: X \to K_a$, $a \in A$. If Λ is an approximate semi-projection of X, then X admits an approximate resolution $\boldsymbol{r} = \{r_c: c \in C\}: X \to (\mathcal{Z}, \mathcal{W}) = \{Z_c, \mathcal{W}_c, r_{cc'}, C\}$ such that each Z_c is some K_a.*

Proof. By (2.19) of [11] X admits an approximate commutative uniform resolution $\boldsymbol{q} = \{q_b: b \in B\}: X \to (\mathcal{Y}, \mathcal{V}) = \{Y_b, \mathcal{V}_b, q_{bb'}, B\}$ of polyhedra Y_b. Moreover, we may assume that $B = (B, <)$ is a cofinite, infinite, antisymmetric, directed set (see (I.3.7) of [15]). Since Λ is an approximate semi-projection of X, for any $b \in B$ there exist an $\alpha(b) \in A$ and a map $h_b: K_{\alpha(b)} \to Y_b$ such that

$$(q_b, h_b f_{\alpha(b)}) < \mathcal{V}_b. \tag{1}$$

Since $\boldsymbol{q}$ is an approximate resolution of X, by (2.1) and (2.5) of [11], it satisfies (R1), (R2) and (R3) for **APOL**. Since $K_{\alpha(b)} \in$ **APOL**, by (R2) for $\boldsymbol{q}$ there exist a $\beta(b) \in B$ and a map $k_b: Y_{\beta(b)} \to K_{\alpha(b)}$ such that $\beta(b) > b$ and

$$(f_{\alpha(b)}, k_b q_{\beta(b)}) < h_b^{-1}\mathcal{V}_b = \{h_b^{-1}V: V \in \mathcal{V}_b\}. \tag{2}$$

Thus by (2)

$$(h_b f_{\alpha(b)}, h_b k_b q_{\beta(b)}) < \mathcal{V}_b. \tag{3}$$

Since $\boldsymbol{q}$ is commutative, $q_b = q_{b\beta(b)} q_{\beta(b)}$, by (1) and (3), $(q_{b\beta(b)} q_{\beta(b)}, h_b k_b q_{\beta(b)}) < \mathrm{st}\, \mathcal{V}_b$. Thus by (R3) for $\boldsymbol{q}$, there exist a $\gamma(b) \in B$ such that $\gamma(b) > \beta(b)$ and

$$(q_{bb'}, h_b k_b q_{\beta(b) b'}) < \mathrm{st}^2 \mathcal{V}_b \quad \text{for any } b' > \gamma(b). \tag{4}$$

Since B is infinite, by (A3) for $(\mathscr{Y}, \mathscr{V})$ there exists a $\delta(b) \in B$ such that $\delta(b) > \gamma(b)$, $\delta(b) \neq b$ and

$$q_{\beta(b)b'}^{-1} k_b^{-1} h_b^{-1} \mathscr{V}_b > \mathscr{V}_{b'} \quad \text{for any } b' > \delta(b). \tag{5}$$

Now, we introduce a new order $<^*$ in B as follows: Let $b, b' \in B$. $b <^* b'$ provided (i) $b = b'$, or (ii) $b \neq b'$ and $\delta(b) < b'$. This new order has the following properties:

$$b <^* \delta(b), \tag{6}$$

$$\text{if } b <^* b' \text{ and } b' < b'', \text{ then } b <^* b''. \tag{7}$$

Since $B = (B, <)$ is a cofinite, infinite, directed set, by (6) and (7) it is easy to show that $B^* = (B, <^*)$ is a cofinite, infinite, directed set.

To define an approximate inverse system $(\mathscr{Z}, \mathscr{W}) = \{Z_b, \mathscr{W}_b, r_{bb'}, B^*\}$ we define terms as follows: For any $b \in B^*$ we put $Z_b = K_{\alpha(b)}$ and $\mathscr{W}_b = h_b^{-1}(\mathrm{st}^2 \mathscr{V}_b) \in \mathrm{Cov}(Z_b)$. For any $b <^* b'$ and $b \neq b'$, we put $r_{bb'} = k_b q_{\beta(b)b'} h_{b'} : Z_{b'} \to Z_b$ and for $b = b'$ we put $r_{bb'} = 1_{Z_b}$.

Claim 1. *$(\mathscr{Z}, \mathscr{W})$ is a uniform approximate inverse system.*

Proof. We need to show the conditions (A1)-(A3) and (AU) in [11]. To prove (A1) take any $b <^* b' <^* b''$. By (4), $(q_{b'b''}, h_{b'} k_{b'} q_{\beta(b')b''}) < \mathrm{st}^2 \mathscr{V}_{b'}$ and thus

$$(q_{b'b''} h_{b''}, h_{b'} k_{b'} q_{\beta(b')b''} h_{b''}) < \mathrm{st}^2 \mathscr{V}_{b'}. \tag{8}$$

By (5), $\mathscr{V}_{b'} < q_{\beta(b)b'}^{-1} k_b^{-1} h_b^{-1} \mathscr{V}_b$ and then $\mathrm{st}^2 \mathscr{V}_{b'} < q_{\beta(b)b'}^{-1} k_b^{-1} h_b^{-1} (\mathrm{st}^2 \mathscr{V}_b)$. Thus by (8) and the commutativity of $(\mathscr{Y}, \mathscr{V})$

$$(k_b q_{\beta(b)b''} h_{b''}, k_b q_{\beta(b)b'} h_{b'} k_{b'} q_{\beta(b')b''} h_{b''}) < h_b^{-1} \mathrm{r}(\mathrm{st}^2 \mathscr{V}_b). \tag{9}$$

(9) means that $(r_{bb''}, r_{bb'} r_{b'b''}) < \mathscr{W}_b$. Hence we have (A1).

To prove (A2) take any $b \in B^*$ and any $\mathscr{W} \in \mathrm{Cov}(Z_b)$. Since $k_b^{-1} \mathscr{W} \in \mathrm{Cov}(Y_{\beta(b)})$, by (A3) for $(\mathscr{Y}, \mathscr{V})$ there exists a $b_0 \in B$ such that $b_0 > \delta(b)$ and

$$q_{\beta(b)b'}^{-1} k_b^{-1} \mathscr{W} > \mathrm{st}^2 \mathscr{V}_{b'} \quad \text{for any } b' > b_0. \tag{10}$$

By (6) and (7), $b <^* b_0$. Take any $b_1, b_2 \in B^*$ with $b_0 <^* b_1 <^* b_2$. By (4), $(q_{b_1 b_2}, h_{b_1} k_{b_1} q_{\beta(b_1) b_2}) < \mathrm{st}^2 \mathscr{V}_{b_1}$ and then

$$(q_{b_1 b_2} h_{b_2}, h_{b_1} k_{b_1} q_{\beta(b_1) b_2} h_{b_2}) < \mathrm{st}^2 \mathscr{V}_{b_1}. \tag{11}$$

By (10), (11) and the commutativity of $(\mathscr{Y}, \mathscr{V})$, $(k_b q_{\beta(b) b_2} h_{b_2}, k_b q_{\beta(b) b_1} h_{b_1} k_{b_1} q_{\beta(b_1) b_2} h_{b_2}) < \mathscr{W}$. This means that $(r_{bb_2}, r_{bb_1} r_{b_1 b_2}) < \mathscr{W}$. Hence we have (A2).

To prove (A3) take any $b \in B^*$ and any $\mathscr{W} \in \mathrm{Cov}(Z_b)$. Since $k_b^{-1} \mathscr{W} \in \mathrm{Cov}(Y_{\beta(b)})$, by (A3) for $(\mathscr{Y}, \mathscr{V})$ there exists a $b_0 \in B$ such that $b_0 > \delta(b)$ and

$$q_{\beta(b)b'}^{-1} k_b^{-1} \mathscr{W} > \mathrm{st}^2 \mathscr{V}_{b'} \quad \text{for any } b' > b_0. \tag{12}$$

By (6) and (7), $b <^* b_0$. Take any $b_1 \in B^*$ with $b_0 <^* b_1$. Since $b_0 < b_1$, by (12), $r_{bb_1}^{-1} \mathscr{W} = h_{b_1}^{-1} q_{\beta(b) b_1}^{-1} k_b^{-1} \mathscr{W} > h_{b_1}^{-1} (\mathrm{st}^2 \mathscr{V}_{b_1}) = \mathscr{W}_{b_1}$. Thus we have (A3).

The condition (AU) follows from (5). Hence we have Claim 1. □

Now we may define an approximate map $\boldsymbol{r}=\{r_b: b\in B^*\}: X\to(\mathscr{Z}, \mathscr{W})$ as follows: For each $b\in B^*$ we put $r_b=k_bq_{\beta(b)}: X\to Z_b$.

Claim 2. *$\boldsymbol{r}$ is an approximate map.*

Proof. We need to show the condition (AS). Take any $b\in B^*$ and any $\mathscr{W}\in\operatorname{Cov}(Z_b)$. By (A3) for $(\mathscr{Y}, \mathscr{V})$, there is a $b_0>\delta(b)$ satisfying (12). Then $b<^* b_0$. Take any $b_1\in B^*$ with $b_0<^* b_1$, Since $b_0<b_1$, by (12),

$$q^{-1}_{\beta(b)b_1}k_b^{-1}\mathscr{W}>\operatorname{st}^2\mathscr{V}_{b_1}. \tag{13}$$

By (4), $(q_{b_1\gamma(b_1)}, h_{b_1}k_{b_1}q_{\beta(b_1)\gamma(b_1)})<\operatorname{st}^2\mathscr{V}_{b_1}$. Then by the commutativity of $\boldsymbol{q}$,

$$(q_{b_1}, h_{b_1}k_{b_1}q_{\beta(b_1)})<\operatorname{st}^2\mathscr{V}_{b_1}. \tag{14}$$

By (13), (14) and the commutativity of $\boldsymbol{q}$,

$$(k_bq_{\beta(b)}, k_bq_{\beta(b)b_1}h_{b_1}k_{b_1}q_{\beta(b_1)})<\mathscr{W}. \tag{15}$$

(15) means that $(r_b, r_{bb_1}r_{b_1})<\mathscr{W}$. Thus we have (AS), i.e., we have Claim 2. □

Claim 3. *$\boldsymbol{r}$ is an approximate resolution.*

Proof. We need to show the conditions (R1) and (R2) for polyhedra. However, it is sufficient to show the conditions (R1)* and (R2)* for polyhedra by (2.6) of [11]. To prove (R1)* take any polyhedron P, any $\mathscr{V}\in\operatorname{Cov}(P)$ and any map $m: X\to P$. Take any $\mathscr{V}'\in\operatorname{Cov}(P)$ such that $\operatorname{st}\mathscr{V}'<\mathscr{V}$. Since $\boldsymbol{q}$ is an approximate resolution, it satisfies (R1)* and (R2)* for polyhedra. By (R1)* for $\boldsymbol{q}$, there exist a $b\in B$ and a map $m_b: Y_b\to P$ such that

$$(m, m_bq_b)<\mathscr{V}'. \tag{16}$$

Since $m_b^{-1}\mathscr{V}'\in\operatorname{Cov}(Y_b)$, by (A3) for $(\mathscr{Y}, \mathscr{V})$, there exists a $b_1>b$ such that

$$q^{-1}_{bb_1}m_b^{-1}\mathscr{V}'>\operatorname{st}^2\mathscr{V}_{b_1}. \tag{17}$$

By (14), (17) and the commutativity of $\boldsymbol{q}$,

$$(m_bq_b, m_bq_{bb_1}h_{b_1}k_{b_1}q_{\beta(b_1)})<\mathscr{V}'. \tag{18}$$

By (16) and (18), $(m, m_bq_{bb_1}h_{b_1}k_{b_1}q_{\beta(b_1)})<\operatorname{st}\mathscr{V}'<\mathscr{V}$. This means that $(m, m_bq_{bb_1}h_{b_1}r_{b_1})<\mathscr{V}$. Thus $\boldsymbol{r}$ satisfies (R1)*.

To prove (R2)* take any polyhedron P and any $\mathscr{V}\in\operatorname{Cov}(P)$. Since $\boldsymbol{q}$ satisfies (R2)*, there exists a $\mathscr{V}'\in\operatorname{Cov}(P)$ satisfying condition (R2)* for $\boldsymbol{q}$. Take any $b\in B^*$ and any maps $f, g: Z_b\to P$ such that

$$(fr_b, gr_b)<\mathscr{V}'. \tag{19}$$

Since $r_b=k_bq_{\beta(b)}$, (19) means that $(fk_bq_{\beta(b)}, gk_bq_{\beta(b)})<\mathscr{V}'$. Thus, by the choice of $\mathscr{V}'$, there exists a $b_1>\beta(b)$ such that

$$(fk_bq_{\beta(b)b_1}, gk_bq_{\beta(b)b_1})<\mathscr{V}. \tag{20}$$

Since B is directed, there exists a $b_2 > b_1$, $\delta(b)$. By (6) and (7), $b <^* b_2$. By (20) and the commutativity of $(\mathcal{Y}, \mathcal{V})$, $(fr_{bb_2}, gr_{bb_2}) < \mathcal{V}$. Thus r satisfies (R2)*. Hence we have Claim 3. □

By definition for each b, $Z_b = K_{\alpha(b)}$. Hence, by Claim 3, we complete the proof of Theorem 5. □

Let X be a space and $\mathcal{U} \in \mathrm{Cov}(X)$. Let $|N(\mathcal{U})|$ be the realization of the nerve $N(\mathcal{U})$ of $\mathcal{U}$ with the CW-topology. Since $\mathcal{U}$ is a normal open covering, we have a canonical map $\varphi_{\mathcal{U}}: X \to |N(\mathcal{U})|$ of X with respect to $\mathcal{U}$. That is, $\varphi_{\mathcal{U}}$ satisfies the condition: $\varphi_{\mathcal{U}}^{-1}(\mathrm{st}(U, N(\mathcal{U})) \subset U$ for any $U \in \mathcal{U}$. Here $\mathrm{st}(U, N(\mathcal{U}))$ is the open star at the vertex U *in* $N(\mathcal{U})$.

Lemma 6. *Let X be a space.*

(i) $\mathrm{SP}(X) = \{\varphi_{\mathcal{U}}: \mathcal{U} \in \mathrm{Cov}(X)\}$ *is an approximate semi-projection of* X.

(ii) *If* $\dim X \leq n$, *then* $\mathrm{SP}^n(X) = \{\varphi_{\mathcal{U}}: \mathcal{U} \in \mathrm{Cov}(X)$ *and* $\mathrm{ord}(\mathcal{U}) \leq n+1\}$ *is an approximate semi-projection of* X.

Proof. We give only a proof of (ii), because we can show (i) in a similar way. Take any polyhedron P, any $\mathcal{V} \in \mathrm{Cov}(P)$ and any map $f: X \to P$. Then there exists a simplicial complex K such that $|K| = P$ and $\mathrm{os}(K) = \{\mathrm{st}(v, K): v$ is a vertex of $K\} < \mathcal{V}$. Since $\dim X \leq n$, there exists a $\mathcal{U} \in \mathrm{Cov}(X)$ such that

$$\mathcal{U} < f^{-1}(\mathrm{os}(K)), \tag{21}$$

$$\mathrm{ord}(\mathcal{U}) \leq n+1. \tag{22}$$

Since $\mathcal{U}$ is a normal covering, there is a canonical map $\varphi_{\mathcal{U}}: X \to |N(\mathcal{U})|$ for $\mathcal{U}$, i.e.,

$$\varphi_{\mathcal{U}}^{-1}(\mathrm{st}(U, |N(\mathcal{U})|) \subset U \quad \text{for any } U \in \mathcal{U}. \tag{23}$$

By (22),

$$\dim |N(\mathcal{U})| \leq n. \tag{24}$$

By (21), there is a function $\psi: N(\mathcal{U})^0 \to K^0$ such that

$$U \subset f^{-1}(\mathrm{st}(\psi(U), K)) \quad \text{for each } U \in N(\mathcal{U})^0. \tag{25}$$

Here K^0 denotes the 0-skeleton of K. We show that

$$\psi \text{ is simplicial.} \tag{26}$$

To prove (26) take any simplex $[U_0, U_1, \ldots, U_m]$ of $N(\mathcal{U})$. By the definition of $N(\mathcal{U})$ and (25),

$$\begin{aligned}\emptyset \neq U_0 \cap \cdots \cap U_m &\subset f^{-1}(\mathrm{st}(\psi(U_0), K)) \cap \cdots \cap f^{-1}(\mathrm{st}(\psi(U_m), K)) \\ &= f^{-1}(\mathrm{st}(\psi(U_0), K) \cap \cdots \cap \mathrm{st}(\psi(U_m), K)).\end{aligned}$$

Thus $\emptyset \neq \mathrm{st}(\psi(U_0), K) \cap \cdots \cap \mathrm{st}(\psi(U_m), K)$ and hence $\psi(U_0), \ldots, \psi(U_m)$ span a simplex of K. Hence we have (26).

By (26), ψ induces a simplicial map $\psi: N(\mathcal{U}) \to K$ and a map $|\psi|: |N(\mathcal{U})| \to |K| = P$. Finally, we show that

$$(|\psi|\varphi_{\mathcal{U}}, f) < \mathcal{V}. \tag{27}$$

To prove (27) take any $x \in X$. Then there is a simplex $[U_0, U_1, \ldots, U_m]$ of $N(\mathcal{U})$ such that $\varphi_{\mathcal{U}}(x) \in \mathrm{Int}([U_0, U_1, \ldots, U_m])$. Thus

$$\varphi_{\mathcal{U}}(x) \in \mathrm{st}(U_i, |N(\mathcal{U})|) \quad \text{for each } i,\ 1 \leq i \leq m. \tag{28}$$

By (26), $|\psi|\varphi_{\mathcal{U}}(x)$ is contained in the simplex spanned by $\psi(U_0), \ldots, \psi(U_m)$. Then

$$|\psi|\varphi_{\mathcal{U}}(x) \in \mathrm{st}(\psi(U_{i_0}), K) \quad \text{for some index } i_0,\ 1 \leq i_0 \leq m. \tag{29}$$

By (28), (23) and (25), $x \in \varphi_{\mathcal{U}}^{-1}(\mathrm{st}(U_{i_0}, |N(\mathcal{U})|) \subset U_{i_0} \subset f^{-1}(\mathrm{st}(\psi(U_{i_0}), K))$, and thus

$$f(x) \in \mathrm{st}(\psi(U_{i_0}), K). \tag{30}$$

By (29) and (30), $(|\psi|\varphi_{\mathcal{U}}, f) < \mathrm{os}(K) < \mathcal{V}$. Hence $\mathrm{SP}^n(X)$ is an approximate semi-projection of X. □

Theorem 7. (i) *Any space X admits an approximate resolution of polyhedra* $|N(\mathcal{U})|$, $\mathcal{U} \in \mathrm{Cov}(X)$.

(ii) *Any space X admits an approximate resolution of polyhedra X_a with* $\dim X_a \leq \dim X$.

Theorem 7 follows from Theorem 5 and Lemma 6.

Lemma 8. *Let $p = \{p_a : a \in A\} : X \to (\mathcal{X}, \mathcal{U}) = \{X_a, \mathcal{U}_a, p_{aa'}, A\}$ be an approximate resolution of a space X. If* $\dim X_a \leq n$ *for all $a \in A$, then* $\dim X \leq n$.

Proof. Take any $\mathcal{U} \in \mathrm{Cov}(X)$. Since p is an approximate resolution, it satisfies (B1), by Theorem (2.8) of [11]. By (B1), there exists an $a \in A$ such that $p_a^{-1}\mathcal{U}_a < \mathcal{U}$. Since $\dim X_a \leq n$, there is a $\mathcal{V} \in \mathrm{Cov}(X_a)$ such that $\mathcal{V} < \mathcal{U}_a$ and $\mathrm{ord}(\mathcal{V}) \leq n+1$. Thus $p_a^{-1}\mathcal{V} < p_a^{-1}\mathcal{U}_a < \mathcal{U}$ and $\mathrm{ord}(p_a^{-1}\mathcal{V}) \leq \mathrm{ord}(\mathcal{V}) \leq n+1$. Hence, $\dim X \leq n$. □

Theorem 1 follows from (ii) of Theorem 7 and Lemma 8. □

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